Solutions to Tutorial 2

Q1). For each process, the relation $Q = \Delta U + W$ holds true. Using this relation for all processes:-

Process	ΔU	Q	W
1-2	610	0	-610
2-3	670	900	230
3-4	-920	0	920
4-1	-360	-360	0

Q2). 2 kg of air is present.

$v_1 = 0.4 \, m^3/kg$, $V_1 = 0.8 m^3$
$T_1 = 650K$

Initial state

$v_2 = 1.1 m^3/kg$, $V_2 = 2.2 m^3$
$T_2 = 310K$.

Final state.

The expansion from the initial state to final state follows the process

$$Pv^{1.7} = 0.8$$ (P is in bar, v is in m^3/kg)

∴ To calculate pressures at initial and final states:-

$P_1 \times 0.4^{1.7} = 0.8$

$\Rightarrow \boxed{P_1 = 3.798 \text{ bar}}$

$P_2 \times 1.1^{1.7} = 0.8 \Rightarrow \boxed{P_2 = 0.68 \text{ bar}}$

Now, for a system which follows a general $PV^n = \text{Constant}$ process, work that is being done can be calculated as:-

$$W = \int_1^2 P\,dV = \int_1^2 k V^{-n}\,dV$$

$$[\text{if } PV^n = k]$$

$$= k \int_1^2 V^{-n}\,dV$$

$$= \left. \frac{k\,V^{-n+1}}{-n+1} \right|_1^2$$

$$= \frac{k}{1-n}\left[V_2^{1-n} - \cancel{P} V_1^{1-n}\right]$$

$$= \frac{kV_2^{1-n} - kV_1^{1-n}}{1-n}$$

$$= \frac{P_2 V_2^n V_2^{1-n} - P_1 V_1^n V_1^{1-n}}{1-n}$$

$$= \frac{P_2 V_2 - P_1 V_1}{1-n} = \text{Specific work.}$$

(work interaction per unit mass)

$\therefore$ In this process given,

$$\text{Work} = m\left(\frac{P_2 V_2 - P_1 V_1}{1-n}\right)$$

$$= 2 \times \frac{\left[(0.68 \times 10^5 \times 2.2) - (3.798 \times 10^5 \times 0.8)\right]}{1-1.7}$$

$$= 440{,}685.7\ \text{J} \approx 440.7\ \text{kJ}$$

Ans

Change in internal energy,

$$\Delta U = m C_v \Delta T$$

$$= 2 \times 718 \, (310 - 650)$$

$$= -488240 \, J$$

$$\approx -488.24 \, kJ$$

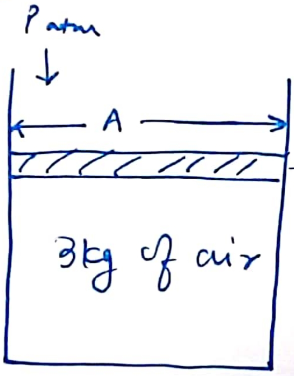
Now,

$$Q = \Delta U + W$$

$$= -488.24 + 440.7$$

$$= -47.54 \, kJ \quad \underline{\underline{Ans}}$$

Q3.



A = area of piston $= 0.01 \, m^2$

40kg mass of piston.

Initial volume of air $= V_i = 0.004 \, m^3$

After a process, volume of air decreases to $0.002 \, m^3$.

$$\therefore V_f = 0.002 \, m^3$$

And simultaneously, 2kJ of heat is lost to the surroundings.

$$\therefore Q = -2000 \, J$$

On the air present inside the cylinder, a ~~con~~ pressure of $P_{atm} + \frac{mg}{A}$ constantly acts.

$$P_{air} = P_{atm} + \frac{mg}{A}$$

$$= 100000 + \frac{40 \times 9.81}{0.01}$$

$$= 139240 \text{ Pa.}$$

The volume decrease (in the air) acts at this constant pressure.

Thus, work interaction

$$W = P_{air}(V_2 - V_1)$$

$$= 139240(0.002 - 0.004)$$

$$= -139240 \times 0.002$$

$$= -278.48 \text{ J}$$

We know that $Q = -2000 \text{ J}$

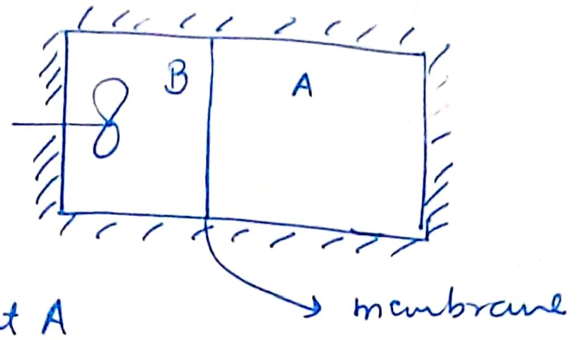
$$\therefore \quad Q = \Delta U + W$$

$$-2000 = \Delta U + (-278.48)$$

$$\Rightarrow \quad \Delta U = -2000 + 278.48$$

$$= -1721.52 \text{ J} \quad \text{Ans}$$

Q4)



Compartment A

Initially evacuated.

$V_{A,i} = 0.4 m^3$ = initial volume.

Compartment B

Initial volume

$V_{B,i} = 0.3 m^3$

Contains 3kg of a pure substance.

($C_v = 700 J/kg/K$

$Pv = 200T$).

Initial pressure

$P_{B,i} = 750 kPa$.

Pure substance is stirred by a fan till the membrane breaks. This happens when

$P_{B,f} = 2.5 MPa = 2500 kPa$

Initially, in B; $Pv = 200T$ (P is in Pa, v is in m^3/kg)

$$\therefore 750 \times 1000 \times \frac{0.3}{3} = 200 \times T$$

$$\therefore T_{B,i} = \frac{750 \times 1000 \times 0.1}{200} = 375 K.$$

Since the volume of compartment B remains constant till the membrane breaks; the initial & final state of B (at which the membrane breaks) are related by:-

$$\frac{P_{B,i}}{T_{B,i}} = \frac{P_{B,f}}{T_{B,f}}$$

$$\therefore \frac{750 \times 1000}{375} = \frac{2.5 \times 10^6}{T_{B,f}}$$

$$\Rightarrow T_{B,f} = 1250\,K \quad \underline{\underline{(a)}}$$

Since the vessel is insulated, $Q = 0$

$$\therefore Q = \Delta U + W$$

$$0 = \Delta U + W$$

$$W = -\Delta U$$

$$= -m\,C_v\,(T_{B,f} - T_{B,i})$$

$$= -3 \times 700\,(1250 - 375)$$

$$= -1837.5\,kJ$$

$\therefore$ The fan does 1837.5 kJ of work on the air. $\underline{\underline{(b)}}$

Since ~~the~~ compartment A is initially evacuated, when the membrane breaks, the gas in Compartment B occupies the whole volume (A + B) instantly. Thus, there are no heat and work interactions after the rupture. Thus, the internal energy & temperature remain constant.

Thus, after rupture,

$$V_f = 0.4 m^3 + 0.3 m^3 = 0.7 m^3$$

$T_f = T_{B,f} = 1250\,K$ $\underline{\underline{(c)}}$

To calculate the equilibrium pressure after the membrane has ruptured, we use the P-V-T relation given in the question.

$$Pv = 200T \quad (P \text{ is in Pa, } v \text{ is in } m^3/kg).$$

$$P_f v_f = 200\, T_f$$

$$v_f = \frac{0.7}{3} \frac{m^3}{kg}.$$

$$\therefore P_f \times \frac{0.7}{3} = 200 \times 1250$$

$$\Rightarrow P_f = 1071.4\ kPa. \quad \underline{\underline{(c)}}$$